NUMERICAL INVESTIGATION OF THE TEMPERATURES AND THERMAL STRESSES IN THERMAL TREATMENT OF METALLIC ARTICLES

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An algorithm is proposed for the numerical computation of the temperatures, stresses, and strains in articles during heat treatment. Specific computations are performed for plates from ShKh15 steel.

In connection with the increased demands on the quality of articles subjected to heat treatment, as well as the necessity to save metals and energy, the problem of a theoretical determination of the temperature and thermal stresses during heat treatment becomes more and more urgent. The difficulties in formulating and solving such a problem are related primarily to the structural transformations in the metal. In the majority of previous papers these transformations were not taken into account, and the results obtained were far from realistic. The first attempt to take account of structural stresses was made in [1-4] within the framework of thermoelasticity theory. The problem of stress analysis during heat treatment with plastic deformations taken into account was examined most fully in [5]. However, the dependences of the thermophysical and mechanical characteristics of the material on the temperature, the heat of the phase transformations, and the kinetic plasticity at the time of the transformations were not taken into account in [5], which could influence the results obtained substantially. Application of numerical methods in recent years permitted the solution of a number of heat-treatment problems of practical importance. The temperature field and structure of an infinite hollow cylinder during hardening was computed in [6]. The papers [7, 8], in which an analysis is given of transformation models and a method of computing the structural components on a thermokinetic diagram (TKD) is presented, are also devoted to determining the temperature fields of articles during hardening with the heat of phase transformation taken into account. Paper [9] is devoted to the numerical computation of stress fields during plate hardening on an electronic computer. An attempt was apparently first made there to take account of the effect of kinetic plasticity during martensite transformations, including the elevated metal plasticity during phase transformations [10], in a stress analysis in a plate being hardened. Unfortunately, the magnitude of the kinetic plasticity coefficient is not indicated and the calculation algorithm is not presented in [9]. This does not permit utilization of the results of this paper in performing analogous computations.

An algorithm and results of a numerical computation of the temperature, stresses, and strains during heat treatment are elucidated in this paper, in which the phase transformations in metals, the temperature dependences of the physical properties, the possibility of the origination of plastic strains and the kinetic plasticity are taken into account. The problem of computing the thermal stresses and strains in the case under consideration can be separated into two problems to be performed sequentially [11]: 1) the computation of the temperature fields and the structural components; and 2) the computation of the thermal stress fields.* An infinite plate was selected as object of investigation in this paper. The temperature problem for an infinite plate can be formulated in the form

$$c(\vartheta)\rho(\vartheta)\frac{\partial\vartheta}{\partial\tau}=\frac{\partial}{\partial z}\left[\lambda(\vartheta)\frac{\partial\vartheta}{\partial z}\right]+Q, \tag{1}$$

$$\lambda(\vartheta)\left.\frac{\partial\vartheta}{\partial z}\right|_{z=0}=\alpha\left(\vartheta|_{z=0}-\vartheta_c\right), \tag{2}$$

*Because of the insufficiency of literature data, the influence of the stresses on the degree of structural transformations was not taken into account in the paper.

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$$-\lambda(\vartheta)\left.\frac{\partial\vartheta}{\partial z}\right|_{z=\delta}=\alpha\left(\vartheta|_{z=\delta}-\vartheta_c\right). \quad (3)$$

The heat liberation Q during phase transformations is determined from the known formula [6, etc.]

$$Q=q\rho\frac{dm}{d\tau}. \quad (4)$$

The computational method in [12], based on the theory of nonisothermal flow with the Mises fluidity condition [13] and our modification taking account of the kinetic plasticity effect, was used in investigating the thermal stress state. On the basis of [12], we write the relation between the strain and stress increments

$$\begin{gathered}\Delta\varepsilon_{xx}=\Psi(\sigma_{xx}-\sigma)+\gamma\sigma-b_{xx},\\ \dots\dots\dots\dots\dots\\ \Delta\varepsilon_{xy}=\Psi\sigma_{xy}-b_{xy},\\ \dots\dots\dots\dots\dots\end{gathered} \quad (5)$$

$$\begin{gathered}b_{xx}=\left(\frac{\sigma_{xx}-\sigma}{2G}\right)^*+(\gamma\sigma)^*-\Delta\varphi,\\ \dots\dots\dots\dots\dots\\ b_{xy}=\left(\frac{\sigma_{xy}}{2G}\right)^*,\\ \dots\dots\dots\dots\dots\\ \Psi=\frac{1}{2G}+\Delta\eta+\Delta F_M.\end{gathered} \quad (6)$$

The quantities with the asterisk refer to the time $\tau-\Delta\tau$, and without the asterisk to the time τ. The coefficient Ψ at the time τ is determined on the basis of the fluidity condition

$$\begin{gathered}\Psi=\frac{1}{2G}+\Delta F_M,\quad \text{if}\quad \sigma_i^2-\sigma_T^2(\vartheta)<0,\\ \Psi=\frac{1}{2G}+\Delta\eta+\Delta F_M,\quad \text{if}\quad \sigma_i^2-\sigma_T^2(\vartheta)=0.\end{gathered} \quad (7)$$

Since the quantities b_{xx}, ..., b_{xy} are known from the initial conditions at $\tau = 0$, then relationships (5) and (6) permit sequential computation of the stresses and strains at each step in time. We now apply the method elucidated above to compute the stresses and strains in the infinite plate being hardened. Since the plate is free of surface forces and the temperature varies only along its thickness, i.e., $\vartheta = \vartheta(z)$, it can then be assumed [11] that under these conditions the stress and strain components will have the form

$$\sigma_{xx}(z)=\sigma_{yy}(z),\ \sigma_{zz}(z)=0,\ \sigma_{xz}=\sigma_{yx}=\sigma_{zy}=0,\ \varepsilon_{xx}(z)=\varepsilon_{yy}(z). \quad (8)$$

From (5) it is not difficult to obtain

$$\sigma_{xx}=\frac{3}{\Psi+2\gamma}(\Delta\varepsilon_{xx}+b_{xx}). \quad (9)$$

Since $\Delta\varepsilon_{xx}$, $\Delta\varepsilon_{yy}$ and $\Delta\varepsilon_{zz}$ are functions of the coordinate z, then from the strain compatibility condition [14] we have an equation for $\Delta\varepsilon_{xx}$:

$$\frac{\partial^2\Delta\varepsilon_{xx}}{\partial z^2}=0. \quad (10)$$

Hence

$$\Delta\varepsilon_{xx}=C_1z+C_2. \quad (11)$$

Now, (9) becomes

$$\sigma_{xx}=\frac{3}{\Psi+2\gamma}(C_1z+C_2+b_{xx}). \quad (12)$$

We find the constants C_1 and C_2 from the equilibrium condition

$$\int_0^\delta \sigma_{xx} dz = 0. \quad (13)$$

$$\int_0^\delta \sigma_{xx}\left(z - \frac{\delta}{2}\right) dz = 0. \quad (14)$$

Taking account of the kinetic plasticity is the most complex problem and not studied in practice. Our kinetic plasticity function F_M for a martensite transformation is taken in the form $\Delta F_M = K\Delta M$, from data in [9], where ΔM is the increment in the degree of martensite structure transformation between the times $\tau - \Delta\tau$ and τ, and K is a proportionality factor. As in [5], the function of the free change in volume in (6) is selected in the form

$$\varphi = P(L_{31} - ML_{21}) + ML_{21} + L_{11}. \quad (15)$$

The coefficients L_{11}, L_{21}, L_{31} equal

$$L_{11} = \frac{V_1(\vartheta) - V_1(\vartheta_0)}{3V_1(\vartheta_0)}, \quad L_{21} = \frac{V_2(\vartheta) - V_1(\vartheta)}{3V_1(\vartheta_0)}, \quad L_{31} = \frac{V_3(\vartheta) - V_1(\vartheta)}{3V_1(\vartheta_0)},$$

where V_1, V_2, V_3 are the specific volumes of austenite, martensite, and a ferrite–cementite mixture, respectively.

The specific volumes of the structural components V_1, V_2, V_3 of steel as a function of the carbon temperature and concentration were calculated by the Yur'ev formulas [15].

A numerical algorithm, realized on an ES type electronic computer, was compiled to compute the temperature fields from (1)-(3) and the stress fields from (12)-(14). An iteration process in Ψ, elucidated in [12], was used in the calculation of the stress and strain with (7) taken into account. Specific computations were performed for plates from the ShKh15 steel (δ = 0.01 m) during hardening in oil. As in [6], the quantities of the structural components were determined by the TKD presented in [16]. The author of [6] covered the TKD domain by a mesh to each of whose nodes a specific structure corresponded. Such a method could apparently result in noticeable error in the calculation of the degree of structure transformation within the characteristic TKD zones. Hence, we took the following method to compute the degree of perlite and bainite transformation. The entry points in each of the transformation domains in temperature and time were determined first. The degree of transformation of each of the structures was determined with the residence time of the computational point in the transformation zone taken into account

$$P = a_P(\tau - \tau_P), \quad (16)$$

$$B = a_B(\tau - \tau_B), \quad (17)$$

where a_P and a_B are factors obtained by miscounting the cooling curves shown on the TKD. Since the dependence of the degree of martensite transformation on the temperature is linear in nature [17], the degree of martensite transformation was computed from the formula

$$M = \frac{\vartheta_M - \vartheta}{\vartheta_M - \vartheta_C}(1 - P - B). \quad (18)$$

There are no data in the literature on the magnitude of K. From a verbal communication of the author of [9], for ShKh15 steel $K \approx 0.001$. In the computations we performed, the values of K were variated in a broad range. The values of the heats of transformation in the temperature field computations are presented in [6], while the thermophysical and mechanical properties of steel ShKh15 are in [18] and [19], respectively. The dependence of the heat transfer coefficient α on the surface temperature was computed from the data in [9].

Computations showed (Fig. 1) that the stresses at the surface are tensile at the initial moments of cooling, and compressive at the center, while they change sign with cooling. These results are qualitatively in agreement with the pattern of the stress change during hardening, known from the literature on heat treatment [17], and also with the computations in [9].

Since there are no exact values of σ_T for steel ShKh15 at high temperatures in the literature, it would be important to estimate the influence of this parameter on the magnitude of the residual stress. Hence, values of σ_T at the hardening temperature 840°C were variated in the computations, and taken equal to 2.5, 5, and 10 kg/mm². The computations showed that

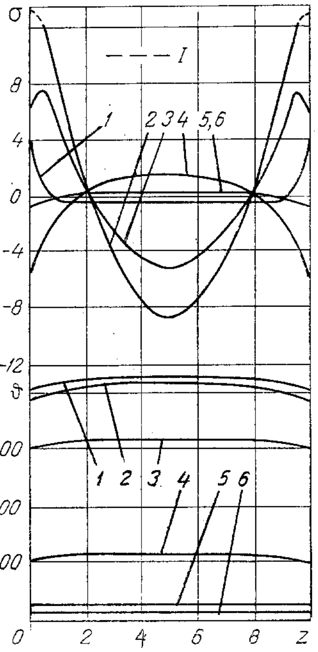

Fig. 1

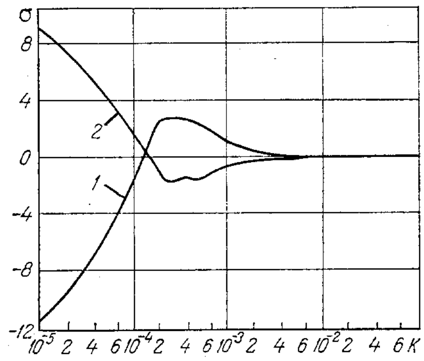

Fig. 2

Fig. 1. Stress and temperature distribution over the thickness of a plate hardened in oil, at different times τ (I is the plasticity zone): 1) τ = 0.01 sec; 2) 0.97; 3) 14.3; 4) 38.1; 5) 190; 6) 1427. σ, kgf/mm²; z, mm; ϑ, °C.

Fig. 2. Dependence of the residual stresses σ in a plate on the kinetic plasticity coefficient K: 1) plate surface; 2) plate center.

the residual stresses in the plate are practically independent of the selection of σ_T in the high temperature range. The coefficient K varies in the 10^{-5}-10^{-1} band. As is seen from Fig. 2, changes in K in the 10^{-5}-10^{-2} range influence the residual stresses substantially.

It is interesting to compare the results obtained with experimental data. There are no test data in the literature on measurements of the strain and stress during plate hardening. Hence, the authors used the results of experiments in [20] in which the beam deflection of steel ShKh15 was measured during hardening under load. The degree of martensite transformation was also determined in the experiments. The diagram of the experimental set-up and the method of measurement are elucidated in [21]. We plotted the data corresponding to the experimental conditions in a computation of the deflection according to the program developed. In the case of the beam $\sigma_{yy} = \sigma_{zz} = 0$ and in place of (9) we obtain on the basis of (5)

$$\sigma_{xx} = \frac{3}{2\Psi + \gamma} (\Delta\varepsilon_{xx} + b_{xx}). \tag{19}$$

Taking account of the bending moment acting on the beam, the equation

$$\int_0^\delta \sigma_{xx} \left(z - \frac{\delta}{2}\right) dz + R = 0 \tag{20}$$

is used in place of (14), where R = Px/2, and x is the coordinate along the beam length.

The beam deflection y was determined by solving the differential equation $y'' = \varkappa(x)$, where $\varkappa(x) = C_1$ is the beam curvature.

The heat-transfer coefficient α as a function of the surface temperature of a jet air-cooled beam was computed from experimental values of the time change in the beam temperature obtained by the author [20]. The results of comparing the test and computed values of the data are presented in Fig. 3.

The initial values of the deflections at M = 0 differ by approximately 2.5 times by computation and experiment. This is apparently associated with the fact that the effective load

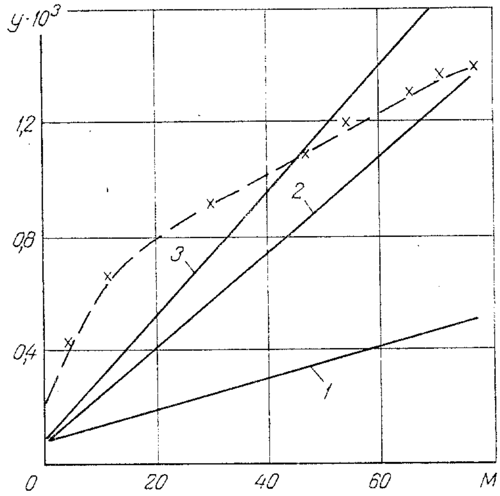

Fig. 3. Dependence of the beam deflection y on the degree of martensite transformation M (dashed line is experimental data [20], and solid line is the computed data): 1) K = 0.001; 2) 0.002; 3) 0.004. $y \cdot 10^3$, m; M, %.

P in the tests [20] could cause stresses exceeding the yield point of the beam material at high temperatures in certain cases. It is seen from Fig. 3 that the order of magnitude of the deflections obtained by experimental and computational means is identical. These data could be brought together by an appropriate change in the kinetic plasticity factor K. The main reason for the discrepancy between the test and computed values is probably the approximate connection between the strains and stresses in the martensite transformation domain plotted in the computations from [9]. At this time further investigations of the kinetic plasticity effect are required which would permit establishment of a more accurate physical connection between the stresses and strains during phase transformations.

On the basis of the investigations it can be assumed that the computation algorithm presented can be utilized in computations of the temperatures, stresses, and strains of articles during heat treatment.

NOTATION

λ, heat conduction; C, specific heat; ρ, density; ϑ, temperature; ϑ_M, temperature of the beginning of martensite transformation; τ, time; τ_P, τ_B, time of the beginning of perlite and bainite transformation, determined by the TKD; α, heat-transfer coefficient; z, coordinate over the plate thickness; δ, plate thickness; q, specific heat of the phase transformation; m, degree of structure transformation; $\varepsilon_{xx}, \ldots, \varepsilon_{xy} \ldots$, strain tensor components; $\sigma_{xx}, \ldots, \sigma_{xy}$, stress tensor components; $\sigma = 1/3(\sigma_{xx} + \sigma_{yy} + \sigma_{zz})$; σ_T, material yield point; η, a proportionality factor between the stress and strain deviators in the plastic zone; G, shear modulus; γ, volume compression modulus; φ, a function of the free change in volume; σ_i, stress intensity; F_M, kinetic plasticity function for the martensite transformation; P, B, M, degrees of perlite, bainite, and martensite transformation, respectively; $\varkappa$, plate curvature; P, force. Subscripts: P, perlite; B, bainite; M, martensite; C, medium; 0, initial value.

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MATHEMATICAL MODEL AND ALGORITHMS FOR AN ELECTRONIC COMPUTER ANALYSIS OF THE HEAT AND MASS TRANSFER IN FREEZING THE SOIL

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An analysis is performed of the selection of a mathematical model of the heat and mass transfer in freezing the soil, and an economical algorithm of its computation on an electronic computer is constructed.

Mathematical models of the heat and mass transfer during freezing disperse media can be separated into two groups [1]: in the first are models with a generalized Stefan-type condition on the moving interface of the thawed and frozen zones, while models without extraction of the freezing front with phase transitions in the whole volume are in the second.

The following assumption is ordinarily made in constructing the mathematical model of the first group: combined heat and mass transfer occurs in the thawed zone, while only heat transfer occurs in the frozen zone. Accordingly, the following system of equations [2] is used for the mathematical description of the freezing process:

$$c_{\text{т}} \frac{\partial T}{\partial t} = \operatorname{div}(\lambda_{\text{т}} \operatorname{grad} T), \tag{1}$$

$$\frac{\partial \omega_1}{\partial t} = \operatorname{div}(k \operatorname{grad} \omega_1), \tag{2}$$

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